ICEN-PS-01/02
February, 2001

Application of the Microlocal Analysis to a Superfield Model in Superspace

Daniel H.T. Franco

Universidade Católica de Petrópolis – (UCP)
Grupo de Física Teórica – (GFT)
Rua Barão do Amazonas 124 - CEP:25685-070 - Petrópolis - RJ - Brasil.

e-mail: daniel@gft.ucp.br

Abstract

In this paper, we apply the microlocal analysis to study the singularity structure of the two-point function in a supersymmetric model formulated in superspace language.

1 Introduction

There are topics, in the physical literature, which do not exhaust themselves, but deserve always new analyzes. Amongst these, the short-distance singularities (the notorious ultraviolet divergences) have a significant part. It is well-known that quantum field theories are deeply connected to the presence of these divergences. Although the renormalization program can overcome this problem in a mathematically proper way, there exists the need for a comprehension of the structure of these singularities. The suitable mathematical framework for this is the wavefront set, introduced by Hörmander and Duistermaat [1, 2] in the seventies for their analysis on the propagation of singularities of pseudodifferential operators.

This subject is of growing importance, with a range of applications going beyond the original problems of linear partial equations. In particular, the link with quantum field theories on a curved spacetime is now firmly established. A short time ago Radzikowski [3], using the notion of wavefront set of a distribution instead of its singular support (which enables to eliminate the difference between local and global results), has generalized a conjecture by Kay [4] that the local Hadamard condition implies the global Hadamard condition. His proof one rely on a general wavefront set spectrum condition for the two-points distribution. Hadamard states are thought to be good candidates for describing physical states, at least for free quantum field theories in curved spacetime, since the work of De Witt and Brehme [5] (see [6, 7, 8] for a general review and references). Thereafter there has appeared a considerable amount of papers devoted to this problem [9]-[14].

At the same time, it seems that not so much attention has been drawn to supersymmetric theories in this direction. Supersymmetry is a subject of considerable interest amongst physicists and mathematicians. It is not only it fascinanting in its own right, but even if 25 years have gone by after its proposal, there exists until a belief that it may play a fundamental role in particle physics. Calculations and phenomenological analysis of supersymmetry models are well-justified in view of the forthcoming generation of machines (NLC and LHC) which shall reveal some of the predicted supersymmetry particles, such as neutralinos, sleptons and may be indirectly squarks. It also has proven to be a tool to link the quantum field theory and non-commutative geometry [15, 16]. Hence, an extension of the technique of wavefront set applied for ordinary quantum field theories to supersymmetric ones seems desirable.

In this work, we will devote special attention to the analysis of the singularity structure of two-point function to a superfield model, characterized in terms of the its wavefront set. Our analysis will be made directly in superspace [17]. Elements of superspace are called supercoordinates which consist of the usual Minkowski spacetime coordinates and anticommuting Grassmann numbers. The concept of superspace was soon realized to represent the appropriate device for a formulation of supersymmetric field theories.

An immediate advantage of the use of superfields is that it renders supersymmetry inherently manifest. Once one knows the action of the supersymmetry transformations in terms of the superspace coordinates, they systematically lead to the transformation laws for the components fields. A further advantage is that superfields automatically accomodate together with the physical fields (those associated with propagating degrees of fredom), a number of unphysical fields, the so-called auxiliary and compensating fields, which play a fundamental role in the formulation of both classical and second quantized supersymmetric field theories.

The organization of this paper is as follows. After this Introduction, we present our free toy model in Section 2. We have confined our attention to an $N = 1/2$-supersymmetric scalar model in two-dimensions for simplicity. In Section 3, we introduce the notion of the microlocal analysis. Section 4 is concerned with a study of the singularity structure to our superfield. In Section 5, we make some concluding remarks and comment on our future perspectives. Finally, the appendix contains some properties of the model which has been employed here.

2 The Free Toy Model

For the sake of simplicity in the presentation, we restrict our discussion to an $N = 1/2$-supersymmetric scalar model in two-dimensions formulated over the $N = 1/2$-superspace parametrized by coordinates (x^{++}, x^{--}, ξ_+),* subjected to the motion equation

$$\partial_{++} d_- \Phi(x, \xi) = 0 \ , \tag{2.1}$$

which is derivable from the free action

$$S_{\text{free}} = \int d^2x d\xi \ \partial_{++}\Phi(x,\xi) d_-\Phi(x,\xi) \ . \tag{2.2}$$

The superfield, $\Phi(x, \xi)$, can be defined in terms of a power expansion of the spinorial variable ξ with x-dependent coefficients, the so-called component fields:

$$\Phi(x, \xi) = \phi(x) + i\xi_+\psi_-(x) \ , \tag{2.3}$$

where $\phi(x)$ is a free boson field and $\psi(x)$ is a free Majorana-Weyl spinor; both massless due to the chiral symmetry. As usual,

$$d_- = \partial_\xi - i\xi_+\partial_{--} \ , \tag{2.4}$$

is a supersymmetric covariant derivative.

To our classical superfield, we may associate a quantum superfield, an operator-valued "superdistribution," smeared with a "supertest" function defined by [18]

$$F_+(x,\xi) = f_+(x) + i\xi_+ f(x) \ , \qquad F(x,\xi) \in \mathcal{D}(\mathbb{R}^{2,1}) = \mathcal{D}(\mathbb{R}^2) \otimes \Lambda C^1 \ , \tag{2.5}$$

*x^{++} and x^{--} are the light-cone coordinates and ξ_+ is a Grassmann coordinate (see appendix).

where ΛC^1 stands for the 1-dimensional space parametrized by the Grassmann coordinate ξ_+. $f(x) \in \mathcal{D}(\mathbb{R}^2)$ is a scalar test function, while $f_+(x) \in \mathcal{D}(\mathbb{R}^2)$ is a spinorial test function.

Remark: The invariance under the Lorentz "charge" requires that the supertest functions have spinorial character. We refer to Apendix for more details on conventions and notations.

For all $F(x,\xi), G(x,\xi) \in \mathcal{D}(\mathbb{R}^{2,1})$, we define the commutation relation

$$[\Phi(F), \Phi(G)] = \int d^2x d^2x' d\xi d\xi' \ \Delta^{\text{susy}}(x,\xi;x',\xi') F(x,\xi) G(x',\xi') \ . \tag{2.6}$$

We call $\Delta^{\text{susy}}(x,\xi;x',\xi')$ the Pauli-Jordan superdistribution, fundamental solution of the operator $\partial_{++}d_-$.

Proposition 2.1 *The two-point function $\Delta^{\text{susy}}(x,\xi;x',\xi')$ has the following form*

$$\Delta^{\text{susy}}(x,\xi;x',\xi') = d_- \left(\Delta(x-x')\delta(\xi_+ - \xi'_+)\right) \ , \tag{2.7}$$

where $\Delta(x-x')$ is the fundamental solution of the Klein-Gordon operator.

Proof. We first observe that the δ-function over the Grassmann variable is defined by

$$\delta(\xi_+ - \xi'_+) = \xi_+ - \xi'_+ \ ,$$

which vanishes for $\xi_+ = \xi'_+$.

Now, using Eqs.(2.4) and (2.5), and then integrating over the ξ_+- and ξ'_+-variables in the Eq.(2.6), we shall get the familiar result in components

$$\begin{aligned}[\Phi(F), \Phi(G)] =& \int d^2x d^2x' \ \{\Delta(x-x') f(x) g(x') - i\partial_{--}\Delta(x-x') f_+(x) g_+(x')\} \\ =& [\varphi(f), \varphi(g)] + \{\psi_-(f_+), \psi_-(g_+)\} \ .\end{aligned}$$

where

$$[\varphi(f), \varphi(g)] = (f, Eg) \ , \quad \{\psi_-(f_+), \psi_-(g_+)\} = (f_+, S g_+) \ ,$$

for all $f, g, f_+, g_+ \in \mathcal{D}(\mathbb{R}^2)$. $E(x,x') \equiv \Delta(x-x')$ is the difference between the advanced and retarded fundamental solution of the Klein-Gordon operator, and

$S(x,x') \equiv -i\partial_{--}\Delta(x-x')$ is the fundamental solution of the Dirac operator. This completes the proof. ∎

Note in particular that due to the proposition 2.1, we get

$$\mathrm{supp}\,\Delta^{\mathrm{susy}}(x,\xi;x',\xi') \subset \mathrm{supp}\,\Delta(x-x') \cup \{\xi_+ \neq \xi'_+\} \ ,$$

with $\mathrm{supp}\,\Delta(x-x') \subset \overline{V}_+ \cup \left(-\overline{V}_+\right)$, where $\overline{V}_+ = \{x \in \mathbb{R}^2 \,|\, x^2 \geq 0,\, x^0 \geq 0\}$ is the future light-cone, being V_+ its interior.

Because we interpret Φ as an operator-valued superdistribution for every supertest function $F(x,\xi) \in \mathcal{D}(\mathbb{R}^{2,1})$, the field equation then may be cast as below:

$$\Phi(\partial_{++}d_-F) = 0 \ , \tag{2.8}$$

and due to the (2.6), we get that $\Delta^{\mathrm{susy}}(x,\xi;x',\xi') \in \mathcal{D}'(\mathbb{R}^{2,1})$ is a fundamental solution which solves the equation

$$\Delta^{\mathrm{susy}}(\partial_{++}d_-F) = 0 \ . \tag{2.9}$$

The vacuum expectation value of the product $\Phi(F)\Phi(G)$ satisfies the relation

$$(\Omega, \Phi(F)\Phi(G)\Omega) = (w_2^{\mathrm{susy}}(x,\xi;x',\xi'),\, F(x,\xi)G(x',\xi')) \ . \tag{2.10}$$

The distribution $w_2^{\mathrm{susy}}(x,\xi;x',\xi')$ extends the Wightman formalism. For this reason, we call $w_2^{\mathrm{susy}}(x,\xi;x',\xi')$ Wightman superdistribution of two-points.

As a consequence of the proposition 2.1, we obtain

$$w_2^{\mathrm{susy}}(x,\xi;x',\xi') = d_-\left(w_2(x-x')\delta(\xi_+ - \xi'_+)\right) \ , \tag{2.11}$$

where $w_2(x-x') = \frac{1}{i}\Delta^{\dagger}(x-x')$, with

$$\Delta^{\dagger}(x-x') = \frac{i}{2\pi}\int d^2k\, \delta(k^2)\theta(k^0)e^{-ik(x-y)} \ . \tag{2.12}$$

The Wightman superdistribution of n-points will be symbolically written under the form [18]

$$w_n^{\mathrm{susy}}\left(x_1,\xi_1;\ldots;x_n,\xi_n\right) = \left(\Omega, \Phi\left(x_1,\xi_1\right)\ldots\Phi\left(x_n,\xi_n\right)\Omega\right) \ , \tag{2.13}$$

and

$$w_n^{\mathrm{susy}}\left(F_n\right) = \int \prod_{i=1}^{n} d^2x_i \prod_{i=1}^{n} d\xi_i \ w_n^{\mathrm{susy}}\left(x_1,\xi_1;\ldots;x_n,\xi_n\right) F_n\left(x_1,\xi_1;\ldots;x_n,\xi_n\right) \ . \tag{2.14}$$

In this definition, we have fixed the order in which we take the distribution and the test function.

3 Briefing on the Microlocal Analysis

The contents of this section can be found in refs. [19]–[23]. We shall introduce the mathematical tool necessary to investigate the distribution singularities, i.e., the wavefront set ($\mathcal{WF}$) of a distribution, a refined description of the singularity spectrum. The main reason for using this tool is that it not only describes the wavefront set of a distribution is singular, but also localize the momenta which constitute these singularities, yielding a simple characterization for the existence of distribution products. Similar notion was developed in other versions by Sato [24], Iagolnitzer [25] and Sjöstrand [26]. The definition as known nowadays is due to Hörmander. He used this terminology due to an existing analogy between his studies on the "propagation" of singularities and the classical construction of propagating waves by Huyghens.

In the classical theory of propagating waves developed by Huyghens, the wave are propagated, for every instant, in a normal direction to the wavefront. In analogy with this theory, for a distribution u we introduce its wavefront set $\mathcal{WF}(u)$ as subset on the momenta space. This subset consists of the points (x, k) for which the direction of the vector k is singular for u in the point x. $\mathcal{WF}(u)$ is independent of the coordinate system chosen, and can be described locally.

The most import point of the Hörmander and Duistermaat analysis, also called microlocal analysis, is to transfer the study of singularities of distributions of the configuration space to the momenta space. For this, we need to "localize" the distribution on the neighborhood of the singularity, examining the result in the Fourier space. The technique consists in multiplying a distribution u for a smooth function ϕ with support contained in a region V, with $\phi(x) \neq 0$, for all $x \in V$. The distribution ϕu can then be seen as a distribution of compact support on $\mathbb{R}^n$. From this point of view, all development is local in the sense that only the behaviour of the distribution on the arbitrarily small neighborhood of the singular point, in the configuration space, is relevant.

As well-known [19, 22] a distribution of compact support, $u \in \mathcal{E}'(\mathbb{R}^n)$, is a smooth function if, and only if, its Fourier transform, $\widehat{u}$, rapidly decreases at infinity. By a fast decay at infinity, one must understanding that for all positive integer N exists a constant C_N such that

$$|\widehat{u}(k)| \leq C_N(1 + |k|)^{-N} < \infty \,, \qquad \forall\, N \in \mathbb{N};\ k \in \mathbb{R}^n \,. \tag{3.15}$$

If, however, $u \in \mathcal{E}'(\mathbb{R}^n)$ is not smooth, then the directions along which $\widehat{u}$ does not fall off sufficiently fast may be adopted to characterize the singularities of u.

Even though a distribution does not have compact support, still we can verify if its Fourier transform rapidly decreases in a given region V, going again through the technique of localization. Its Fourier transform will be defined as a distribution on $\mathbb{R}^n$, and will satisfy the property (3.15).

Definition 3.1 *Let $u \in \mathcal{D}'(\mathbb{R}^n)$ be a distribution and $\phi \in C_0^\infty(V)$ a smooth function with support $V \subset \mathbb{R}^n$. Then, ϕu has compact support.*

The Fourier transform of ϕu produces a smooth function in the momenta space.

Lemma 3.2 *Consider $u \in \mathcal{D}'(\mathbb{R}^n)$ and $\phi \in C_0^\infty(V)$. Then*

$$\widehat{\phi u}(k) = u(\phi e^{-ikx}) \ .$$

Moreover, the restriction of u to $V \subset \mathbb{R}^n$ is asymptotically limited for $k \to \infty$ if, and only if, for every $\phi \in C_0^\infty(V)$ exists a constant $C_{\phi,N}$, such that

$$|\widehat{\phi u}(k)| \leq C_{\phi,N}(1+|k|)^{-N} < \infty \,, \qquad \forall\, N \in \mathbb{N};\ k \in \mathbb{R}^n \ .$$

If $u \in \mathcal{D}'(\mathbb{R}^n)$ is singular in x, and $\phi \in C_0^\infty(V)$ is $\phi(x) \neq 0$; then ϕu is singular in x and has compact support. In some directions $\widehat{\phi u}$ until will be asymptotically limited. This is called the set of *regular directions* of u.

Definition 3.3 *Consider $u(x) \in \mathcal{D}'(\mathbb{R}^n)$. The pair $(x,k) \in \mathbb{R}^n \times (\mathbb{R}^n \backslash 0)$ is called a point describing a regular direction for high momenta to $u(x)$ if, and only if, there exists a neighborhood V of x, a conic neighborhood M of k, and a function $\phi(x) \in C_0^\infty(V)$, with $\phi(x) \neq 0$, such that the Fourier transform of ϕu is asymptotically limited for $k \to \infty$, i.e.,*

$$|\widehat{\phi u}(k)| \leq C_{\phi,N}(1+|k|)^{-N} < \infty \qquad \forall N \in \mathbb{N};\ k \in \mathbb{R}^n \ , \tag{3.16}$$

where $C_{\phi,N}$ are constants. The wavefront set $\mathcal{WF}(u)$ of the distribution $u(x)$ consists of the pairs $(x,k) \in \mathbb{R}^n \times (\mathbb{R}^n \backslash 0)$, of points x in the configuration space and k in the Fourier space, such that the Fourier transform $\widehat{\phi u}$ not decay sufficiently rapid along of the direction k, for $|k| \to \infty$.

The wavefront set $\mathcal{WF}(u)$ is conic in the sense that it remains invariant under the action of dilatations, i.e. when we multiply the second variable by a positive scalar. This means that if $k \in \mathcal{WF}(u)$ then $\lambda k \in \mathcal{WF}(u)$ for all $\lambda > 0$. Thus, (x, k) is a point describing a regular direction if its "localization" ϕu in a small neighborhood of x has Fourier transform decreasing sufficiently fast for any power in a cone around k.

From the definition of $\mathcal{WF}(u)$ for $u \in \mathcal{D}'(\mathbb{R}^n)$ it follows that the projection $\pi_1(\mathcal{WF}(u)) \to x$ is the singular support of u. Roughly speaking, if $(x, k) \in \mathcal{WF}(u)$ then x specifies the localization of a singularity of u and k its "direction of propagation"

$$u \text{ is singular in } x \quad \Longleftrightarrow \quad \exists\, k \in \mathbb{R}^n \backslash 0 \mid (x, k) \in \mathcal{WF}(u) \ .$$

The projection onto the second variable is represents by $\pi_2(\mathcal{WF}(u)) \to \Sigma(u)$, where $\Sigma(u)$ is defined to be the complement in $\mathbb{R}^n \backslash 0$ of the set of all $k \in \mathbb{R}^n \backslash 0$ for which there is an open conic neighborhood M of k such that $\widehat{\phi u}$ if of rapid decrease in M.

We emphasize that, as the notion of the wavefront set applies to distributions, it can be used to theories which are formulated in terms of pointlike fields. In the naive perturbative scheme of quantum field theories, one encounters formal products of fields which are a priori not well-defined. The ultraviolet problems involved in defining the product of these fields, can be conveniently controlled by the so-called **Hörmander Criterium**:

Theorem 3.4 (Theorem IX.45 in [19]) *Let u and v be distributions. Suppose that*

$$(x, 0) \notin \mathcal{WF}(u) \oplus \mathcal{WF}(v) = \{(x, k_1 + k_2) \mid (x, k_1) \in \mathcal{WF}(u), (x, k_2) \in \mathcal{WF}(v)\} \ .$$

Then, the product uv exists and

$$\mathcal{WF}(uv) \subset \mathcal{WF}(u) \cup \mathcal{WF}(v) \cup (\mathcal{WF}(u) \oplus \mathcal{WF}(v)) \ .$$

Hence, the product of distributions u e v is well-defined, in x, if u, or v, or both distributions are regulars in x. If u and v are singulars in x, the product exists if the sum of second component of the $\mathcal{WF}(u)$ and $\mathcal{WF}(v)$ in x is different of zero.

Another important general fact is that the multiplication with a smooth function and differentiation do not enlarge the wavefront set:

$$\mathcal{WF}(au) \subset \mathcal{WF}(u) \quad \text{if } a \in C^\infty \ , \tag{3.17}$$

$$\mathcal{WF}(Pu) \subset \mathcal{WF}(u) \quad \text{if } P \text{ is any linear differential operator} \ . \tag{3.18}$$

4 Singularity Structure of the Two-Point Function of the Superfield $\Phi(x, \xi)$

To explore the notion of the microlocal analysis of singularities, we need a method to compute the wavefront set of a distribution. For this, we will go through another important tool developed by Hörmander, the so-called Fourier Integral Distribution, or Oscillatory Integral, employed in the study of pseudodifferential operators. Pseudodifferential operators allow one to give an alternative and more natural definition of the wavefront set. Here, following the presentation of ref. [23], we shall use the stationary phase method, which appears in the development of the theory of pseudodifferential operators in order to find the asymptotic behaviour of an integral of the form $\int dx\ e^{i\lambda\varphi(x)}a(x)$, when $\lambda \to \infty$ and φ has critical points.

Pseudodifferential operators generalize linear differential operators with variable coefficients. If $p(x, D) = \sum_{|\alpha \leq m|} a_\alpha(x) D_x^\alpha$ is a differential operator with x-dependent coefficients, then

$$\begin{aligned} p(x, D)u(x) &= \frac{1}{(2\pi)^n} p(x, D) \int_{\mathbb{R}^n} d^n k\ e^{ikx} \widehat{u}(k) \\ &= \frac{1}{(2\pi)^n} \int_{\mathbb{R}^n} d^n k\ p(x, k) e^{ikx} \widehat{u}(k) \ , \end{aligned} \tag{4.19}$$

where $u(x) \in \mathcal{D}(\mathbb{R}^n)$, $\widehat{u}(k)$ is the Fourier transform, $p(x, k) = \sum_{|\alpha \leq m|} a_\alpha(x) k^\alpha$. Replacing $p(x, k)$ by appropriate functions, called *symbols*, we obtain a pseudodifferential operator. The symbols is nothing else, in this case, but the polynomial $p(x, k)$ obtained by substituting the variable k_j for the partial differentiations D_j.

Definition 4.1 *For an open set $X \subset \mathbb{R}^n$, and m, ρ, δ real numbers, with $0 < \rho \leq 1$ and $0 \leq \delta < 1$; one define the symbol space $S^m_{\rho,\delta}(X \times \mathbb{R}^s)$, on $X \times \mathbb{R}^s$, of* **order** *m and* **type** *(ρ, δ), as being the space of smooth functions $a(x, k)$, such that for any* *compact set $\Omega \subset X$, where the functions $a(x,k)$ taking their values, and multi-indices $\alpha \in \mathbb{N}^n, \beta \in \mathbb{N}^s$, exists a constant $C_{\alpha,\beta,\Omega}$ such that*

$$\left| D_x^\alpha D_k^\beta a(x,k) \right| \leq C_{\alpha,\beta,\Omega} (1+|k|)^{m-\rho|\beta|+\delta|\alpha|} \quad \forall\, x \in \Omega; k \in \mathbb{R}^s \ . \tag{4.20}$$

The better constants $C_{\alpha,\beta,\Omega}$, in (4.20) are semi-norms

$$\|a\|_{\alpha,\beta,\Omega} = \sup_{x\in\Omega;\, k\in\mathbb{R}^s} (1+|k|)^{\rho|\beta|-\delta|\alpha|-m} \left| D_x^\alpha D_k^\beta a(x,k) \right| \ . \tag{4.21}$$

Definition 4.2 *Given a symbol $a(x,y,k)$ in $S^m_{\rho,\delta}(X \times X \times \mathbb{R}^s)$, where the variable $k \in \mathbb{R}^s$ is the dual of $x_i \in X$, the pseudodifferencial operator is a Fourier integral operator*

$$Au(x) = \frac{1}{(2\pi)^n} \int d^n k d^n y \ e^{ik(x-y)} a(x,y,k) u(y) \quad \forall\, u \in \mathcal{D}(X) \ . \tag{4.22}$$

We denote by $L^m_{\rho,\delta}(X)$ the space of these operators, and we say that $A \in L^m_{\rho,\delta}(X)$ is of order $\leq m$ and of type (ρ, δ).

In the physical applications of interest, it is sufficient to pay attention to the subclass of symbols $S^m_{1,0}$ first studied by Kohn and Nirenberg [27]. A polynomial with respect to k of degree m, with constant coefficients is of course a symbol $S^m_{1,0}$.

Example. The inverse of $(1-\Delta) : \mathcal{S}(\mathbb{R}^n) \to \mathcal{S}(\mathbb{R}^n)$, where $\Delta = \frac{\partial^2}{\partial x_1^2} + \cdots + \frac{\partial^2}{\partial x_n^2}$ is the Laplacian operator, is given by

$$Au(x) = (1-\Delta)^{-1} = \frac{1}{(2\pi)^n} \int d^n k \ e^{ikx} \frac{1}{(1+k^2)} \widehat{u}(k) \ , \tag{4.23}$$

with $k^2 = \sum_{i=1}^n k_i^2$. Hence, $A \in L^{-2}_{1,0}(\mathbb{R}^n)$.

Example. If A is a differential operator of order $\leq m$ on $X \subset \mathbb{R}^n$ with smooth coefficients, then $A \in L^m_{1,0}(X)$. In fact, if $A = \sum_{|\alpha|\leq m} a_\alpha(x) D_x^\alpha$, with $a \in C^\infty(X)$, then by inverse Fourier transform, we obtain

$$Au(x) = \frac{1}{(2\pi)^n} \int d^n k \ e^{ikx} a(x,k) \widehat{u}(k) = \frac{1}{(2\pi)^n} \int d^n k d^n y \ e^{ik(x-y)} a(x,k) u(y) \ , \tag{4.24}$$

where $a(x,k) = \sum_{|\alpha|\leq m} a_\alpha(x) k^\alpha \in S^m_{1,0}(X \times \mathbb{R}^s)$.

In the example above, the kernel of A is given by an integral so-called *oscillatory integral*

$$K_A(x,y) = \frac{1}{(2\pi)^n} \int d^n k \ e^{ik(x-y)} a(x,k) \ . \tag{4.25}$$

Definition 4.3 *The oscillatory integral – or Fourier integral distribution – on* $X \times \mathbb{R}^s$ *is formally written as*

$$I_\varphi(a) = \int dk \; e^{i\varphi(x,k)} a(x,k) \;, \tag{4.26}$$

where $\varphi(x,k)$ *is a* **phase function** *and* $a(x,k)$ *is an asymptotic symbol.*

An important example of an oscillatory integral is the integral

$$\int_{\mathbb{R}^n} dk \; e^{-ikx} = \delta(x)(2\pi)^n \;,$$

which defines the Dirac's distribution δ.

Definition 4.4 *Let* $X \subset \mathbb{R}^n$ *be open and* Γ *an open cone in* $X \times \mathbb{R}^s \backslash 0$*. This means that* Γ *is invariant if the second component in* $\mathbb{R}^s$ *is multiplyed by positive scalars. We say that the function* $\varphi(x,k) \in C^\infty(\Gamma)$ *is a phase function in* Γ *if*

1. φ *is homogeneous of degree 1 in* k*, i.e.,* $\varphi(x, \lambda k) = \lambda \varphi(x,k)$ *if* $(x,k) \in \Gamma \quad \forall\, \lambda > 0$*;*
2. $\operatorname{Im} \varphi(x,k) \geq 0$*;*
3. $d\varphi = \sum_{i=1}^n \frac{\partial \varphi}{\partial x_i} dx_i + \sum_{i=j}^s \frac{\partial \varphi}{\partial k_j} dk_j \neq 0$*, i.e.,* φ *has no critical points in* Γ*. This means that at every point in* Γ*, some* $\frac{\partial \varphi}{\partial x_i}$ *or* $\frac{\partial \varphi}{\partial k_j}$ *is non-vanishing.*

Definition 4.5 *If* $\varphi \in C^\infty(X \times \mathbb{R}^s \backslash 0)$ *is a phase function, we call*

$$\mathcal{C}_\varphi = \{(x,k) \in X \times \mathbb{R}^s \backslash 0 \mid \varphi'_k(x,k) = 0\} \;,$$

the critical set of φ*. Here,* $\varphi'_k(x,k) = \left(\frac{\partial \varphi}{\partial k_1}, \dots, \frac{\partial \varphi}{\partial k_s}\right)$*. The manifold of stationary phase is the point set*

$$\Lambda_\varphi = \{(x, \varphi'_x(x,k)) \mid (x,k) \in \mathcal{C}_\varphi; k \neq 0\} \;,$$

with, $\varphi'_x(x,k) = \left(\frac{\partial \varphi}{\partial x_1}, \dots, \frac{\partial \varphi}{\partial x_n}\right)$*.*

It is the behaviour of $a(x,k)$ and $\varphi(x,k)$ near $\mathcal{C}_\varphi$ which determines the singularities of $I_\varphi(a)$.

Lemma 4.6 (Lemma 3 in [19], pg. 101) *Λ_φ is a closed subset of $(X \times \mathbb{R}^s \backslash 0)$ and if $(x, k) \in \Lambda_\varphi$, then $(x, \lambda k) \in \Lambda_\varphi$ for all $\lambda \in \mathbb{R}_+$.*

Proposition 4.7 *If $\varphi(x, k)$ is a phase function on $X \times \mathbb{R}^s \backslash 0$ and $a(x, k) \in S^m_{\rho,\delta}(X \times \mathbb{R}^s \backslash 0)$, with $\delta < 1$, $\rho > 0$; then $\mathcal{WF}(I_\varphi(a)) \subset \Lambda_\varphi$.*

Before proving proposition 4.7, it is suitable we recall a few additional results.

Lemma 4.8 *Let $X \subset \mathbb{R}^n$ be an open set, and $u \in C_0^\infty(X)$. If $\varphi \in C^\infty(X)$ is a phase function such that $\operatorname{Im} \varphi \geq 0$ and $d\varphi \neq 0$, i.e., φ has no critical points on the support of u, then the integral*

$$I(\lambda) = \int dx \; e^{i\lambda\varphi(x)} u(x) \; ,$$

rapidly decreases when $\lambda \to \infty$.

The stationary phase method applies when $d\varphi$ is allowed to vanish, but instead one makes the hypothesis that all the critical points of φ are nondegenerate which means that, if $d\varphi(x_0) = 0$, the Hessian matrix of φ at x_0, $\left(\frac{\partial^2 \varphi(x)}{\partial x_j \partial x_k}\right)_{1 \leq j,k \leq n}$, is necessary nonsingular [20, 23]. Hence, if $\varphi \in C^\infty(X)$ is such that $\operatorname{Im} \varphi \geq 0$ and $u \in C_0^\infty(X)$, the asymptotic behaviour of $I(\lambda) \to \infty$ is determined by φ and u, in the neighborhood of the set of critical points of φ, i.e., when $d\varphi = 0$. Thus, the *essential* contributions must always come from the points where the phase φ is real and stationary.

Proof of the proposition 4.7. We first assume that $(x, k) \in (X \times \mathbb{R}^s \backslash 0) \backslash \Lambda_\varphi$. Now, we consider $u \in C_0^\infty(\Omega)$, where $\Omega \subset X$ represents a compact set, with $u(x) \neq 0$ for all $x \in \Omega$. Then, the integral

$$\widehat{I_\varphi(au)}(p) = \int dk dx \; e^{i\widetilde{\varphi}(x,k,p)} a(x, k) u(x) \; , \quad \widetilde{\varphi}(x, k, p) = \varphi(x, k) - xp \; ,$$

must rapidly decrease in the conic neighborhood V of k for all $p \in V$. In order to prove this, we apply the method of stationary phase. We put $p = \lambda p'$ and perform the change of variables $k \to \lambda k'$, such that with $p, k \in V$ also p' and k' are contained in V for all $\lambda > 0$. So we obtain

$$\widehat{I_\varphi(au)}(\lambda p) = \lambda^{-n} \int dk dx \; e^{i\widetilde{\varphi}(x,\lambda k,\lambda p)} a(x, \lambda k) u(x) \; ,$$

dropping by convenience the $'$.

Using the homogeneity of the phase function, then $\widetilde{\varphi}(x,\lambda k,\lambda p) = \lambda\widetilde{\varphi}(x,k,p)$ if (x,k,p) belongs to an open cone Γ in $(X\times\mathbb{R}^s\backslash 0)$ for all $\lambda > 0$. By Lemma 5 (pg.105) in [19], there exists a differential operator, L, such that ${}^tL\, e^{i\lambda\widetilde{\varphi}} = e^{i\lambda\widetilde{\varphi}}$, (where ${}^tL$ is its adjoint) whose coefficients are homogeneous of degree -1 in k,p. This operator is given by

$$L = \frac{1}{i\lambda\Phi(x,k,p)}\left(\frac{\partial\varphi(x,k)}{\partial x_i} - p_i\right) D_{x_i}\ , \quad \Phi(x,k,p) = \left(\frac{\partial\varphi(x,k)}{\partial x_i} - p_i\right)^2 \ .$$

Consequently, we obtain

$$\begin{aligned}\widehat{I_\varphi(au)}(\lambda p) =& \lambda^{-n}\int dkdx\ ({}^tL)^\alpha\, e^{i\lambda\widetilde{\varphi}} a(x,\lambda k)u(x)\\ =& \lambda^{-n}\int dkdx\ e^{i\lambda\widetilde{\varphi}}\, L^\alpha\left(a(x,\lambda k)u(x)\right)\ .\end{aligned}$$

Hence, we get the estimate:

$$\begin{aligned}\left|\widehat{I_\varphi(au)}\right| =& \left|\lambda^{-n-|\alpha|}\int dkdx\ \frac{i^{|\alpha|}e^{i\lambda(\widetilde{\varphi}(x,k,p))}}{\Phi^\alpha(x,k,p)}\left(\frac{\partial\varphi(x,k)}{\partial x_i} - p_i\right)^\alpha D^\alpha_{x_i}\left(a(x,\lambda k)u(x)\right)\right|\\ \leq& \sum_{|\alpha|\leq N} C_{\alpha,\varphi,\Omega}\left(\sup_{x\in\Omega}|D^\alpha u(x)|\right)(1+|\lambda|)^{m-(1-\delta)|\alpha|-n}\ ,\end{aligned}$$

which rapidly decreases if $m-n-(1-\delta)|\alpha| < 0$ to $\lambda\to\infty$. By hypothesis, as $\delta < 1$ and since $|\alpha|$ can be made arbitrarily large, then $I_\varphi(au)$ is asymptotically limited for $\lambda\to\infty$ for an open cone in $(X\times\mathbb{R}^s\backslash 0)$. If m can be chosen arbitrarily negative, $I_\varphi(au)$ is asymptotically limited even for $|\alpha| = 0$. As a result of this, we deduce that $(x,k)\notin \mathcal{WF}(I_\varphi(a))$, which completes the proof. ∎

We are finally ready to state our main result:

Proposition 4.9 *The two-points function $w_2^{\rm susy}(x,\xi;x',\xi')$ of the free massless superfield $\Phi(x,\xi)$ has its wavefront set given by:*

$$\mathcal{WF}(w_2^{\rm susy}) \subset \mathcal{WF}(w_2)\ ,$$

with

$$\mathcal{WF}(w_2) = \{(x,k_1),(x',k_2) \in (\mathbb{R}^2 \times \mathbb{R}^2\backslash 0) \,|\, x \neq x'; (x-x')^2 = 0; k_1||(x-x');$$
$$k_1 + k_2 = 0; k_1^0 \geq 0\} \cup \{(x,k_1),(x',k_2) \in (\mathbb{R}^2 \times \mathbb{R}^2\backslash 0) \,|\, x = x';$$
$$k_1 + k_2 = 0; k_1^2 = 0; k_1^0 \geq 0\} \ .$$

Proof. We first observe that by "Ectoplasmic Integration Theorem" of Gates [28], the topology of a supermanifold must be generated essentially from its bosonic submanifold. So the Grassmannian sector of superspace cannot produce an effect on the singular structure of the two-points $w_2^{\text{susy}}(x,\xi;x',\xi')$. Now, using the representation of $w_2^{\text{susy}}(x,\xi;x',\xi')$, eq.(2.11), and exploring the fact that a differential operator decrease the wavefront set, we are allowed to conclude that

$$\mathcal{WF}(w_2^{\text{susy}}) \subset \mathcal{WF}(w_2) \ . \tag{4.27}$$

The proof then follows that of the Theorem IX.48 of [19] with $w_2(x,x')$ given by following representation of the Fourier transform

$$\widehat{w_2}(x,x') = \frac{1}{2\pi}\delta(k_1+k_2)\theta(k_1^0)\delta(k_1^2) \in \mathcal{D}'(\mathbb{R}^2 \times \mathbb{R}^2) \ .$$

This completes the proof. ∎

Remark: The proposition 4.9 provides us with a "global" wavefront set. In our setting the word "global" means that the singular support of all component fields is embodied in eq.(4.27). Moreover, it reflects the fact that the fundamental solution of the operator $\partial_{++}d_-$ is singular on the light-cone.

That the bosonic sector is responsible by carrying all singular structure of the superspace is not too surprising. Being the key idea of the Hörmander and Duistermaat analysis the shift of the study of singularities of the configuration space to the Fourier space, we recall that by convention the Fourier transform of any object written in superspace language, it is realized only in bosonic sector of the superspace. Moreover, apparently, there no exist reason to have superspaces where the topological properties of the superspace are substantially different from the bosonic submanifold it contains, otherwise this should not be consistent with the usual components-by-projection technique.

The singularity structure of the Feynman free superpropagator can be investigated in the same way. In fact, in theory of the free quantum superfield, one defines the Feynman free superpropagator by

$$i\Delta_F^{\mathrm{susy}}(x,\xi;x',\xi') = w_2^{\mathrm{susy}}(x,\xi;x',\xi') + \Delta_{\mathrm{ret}}^{\mathrm{susy}}(x,\xi;x',\xi') \ .$$

To end up, it is worthwhile to emphasize that a generalization to a more general superfield is straightforward, up to possible (but surmountable) conceptual problems existing in microlocal analysis of a superfield whose component fields are of the multi-component type. In fact, in the case of a more general superfield model whose component fields contain spinorial and/or vectorial fields of the multi-component type, the "global" wavefront set does not contain any information about the components of the component fields that are singular. Hence, although the superspace methods display advantages over components approaches to supersymmetry, making quantum calculations easy, it seems desirable to go through the component expansions of superfield and superspace if we wish find out the singular components of spinorial and/or vectorial fields. This will require the notion of the polarization set $\mathcal{WF}_{\mathrm{pol.}}$, a refinement of the wavefront set, introduced by Dencker [29] in order to analyze the singularities of multi-component distributions. For example, if $u \in \mathcal{D}'(X, \mathbb{C}^N)$ is a multi-component distribution on X, taking their values in $\mathbb{C}^N$; then $u = u_j$, $j = 1, \ldots, N$, where $u_j \in \mathcal{D}'(X, \mathbb{C}^N)$, so that

$$\mathcal{WF}(u) = \bigcup_{j=1}^{n} \mathcal{WF}(u_j) \ . \tag{4.28}$$

As our "global" wavefront set, the wavefront set so defined does not contain any information about the components of the distribution u that are singular. It is the Dencker polarization set which allows us to identify the singular components of the multi-component distribution u (cf. [30, 31, 32] for recent applications of this idea).

5 Concluding Remarks and Outlook

The main purpose of this paper is an attempt to apply the microlocal analysis to study the singularity structure of the two-point function for a superfield model directly in superspace. As mentioned in the Introduction, the concept of superspace was soon realized to represent the appropriate device for a formulation of supersymmetric field theories. This has an advantage of rendering supersymmetry manifest and, moreover, the formalism accomodate together with physical fields, the auxiliary and compensating fields needed for the formulation of supersymmetric field theories.

In spite of the hard works which have been made for a comprehension of the quantization of supersymmetric theories via the formalism of Feynman supergraphics and superpropagators[†] [34], we think that the use of the microlocal analysis to the study of the singularity structure of the superpropagators might refine our understanding of the source of its divergences. This will may contribute significatively to better understanding of interacting supersymmetric quantum field theories.

On the other hand, the inclusion of the gravitation in this scenario remains an open problem of Physics and an active area of current research. Although a significative progress in the energy scale has been reached, the Planck scale ($10^{19}Gev$) (at which effects from quantum gravity are expected to become important) remains unaccessible. From the purely theoretical point of view, all the attempts to include gravity in the quantization program failed up to now. Alternative proposals such as Supergravity, Kaluza-Klein [35] and String theories [36], and more recently the D-brane theory [37] and the Baez-Rovelli formulation [38, 39], have elucidated the role of quantum gravity, without, however, providing conclusive results. For this reason, and because of relevant scale for the MSSM (10^3Gev), a reasonable approximation should be to consider the interaction of matter and gravitational fields as a quantum field theory in curved spacetimes. The gravitational field is included as a background field and the matter fields are quantized as operator-valued Wightman fields. This framework has a wide range of physical applicability, the most prominent being the gravitational effect of particle creation in the vicinity of black-holes, learned about for the first time by Hawking [40].

From an axiomatic point of view, whereas the most of the Wightman axioms can be implemented on a curved background spacetime $\mathcal{M}$, the spectrum condition (which expresses the positivity of the energy) represents a serious conceptual problem. While the Poincaré covariance, in particular the translations, guarantees the positivity of the spectrum, and fixes a unique vacuum state if $\mathcal{M} = \mathbb{R}^4$ is the Minkowski space, this familiar concept of field theory does not exist in a generic

[†]A comprehensive account of the quantum theory through the algebraic renormalization approach can be found in the textbook of Piguet and Sibold [33].

curved background spacetime. So, in general, no useful notion of a vacuum state (or equivalently of a particle interpretation) exist, too.

An advice on how to define the spectrum condition, at least for free quantum field theory in curved spacetime, was given by Wald [41] for purpose of finding the expectation value of the energy-momentum tensor. For free fields, this approach led to the concept of a Hadamard state. The import discovery by Radzikowski [3] that the global Hadamard condition can be locally characterized in terms of the wavefront set, has made the connection with the spectrum condition much more transparent. After this, considerable advances have been made in this direction, especially by Hamburg group. Motivated by insights from these recent advances, and as a very interesting matter for an investigation, we intend to understand how to describe the Hadamard condition directly in superspace. As a example, we investigate the Hadamard condition for the Wess-Zumino model [42] (cf. [9] for an analysis in components). As a next step, we intend to study the renormalization of this model on a "supercurved" background [43] a la Brunetti-Fredenhagen [13].

Acknowledgements

We would like to thank J.A. Helayël-Neto for helpful discussions and illuminating comments on supersymmetry. We are also grateful to the staff of the Group of Theoretical Physics at UCP for the warm hospitality and for the pleasant atmosphere.

A Notations and Conventions

For sake of completeness, we quote some properties of the model which has been employed here. This material is included to render the paper as self-contained as possible.

In two-dimensional theories, a coordinate system much used is the light-cone coordinate one, defined by

$$x^{++} = \frac{x^0 + x^1}{\sqrt{2}} \ , \qquad x^{--} = \frac{x^0 - x^1}{\sqrt{2}} \ . \tag{A.1}$$

Taking into account that the two-dimensional Minkowski space has a metric given by $\eta_{\mu\nu} = \text{diag.}(1, -1)$, one can show that the line element in light-cone coordinates assumes the form

$$ds^2 = 2dx^{++}dx^{--} \ , \tag{A.2}$$

indicating which the metric tensor in this system is given by

$$\eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ . \tag{A.3}$$

Therefore, we get

$$x^{++} = x_{--} \ , \qquad x^{--} = x_{++} \ .$$

An immediate advantage of the use of the light-cone coordinates is that, they do not mix under Lorentz transformations:

$$\begin{pmatrix} \tilde{x}^{++} \\ \tilde{x}^{--} \end{pmatrix} = \begin{pmatrix} e^{\alpha} & 0 \\ 0 & e^{-\alpha} \end{pmatrix} \begin{pmatrix} x^{++} \\ x^{--} \end{pmatrix} \ , \tag{A.4}$$

where α is a parameter of $SO(1,1)$. We say that x^{++} and x^{--} have Lorentz "charges" $+1$ and -1, while that x_{++} and x_{--} have Lorentz "charges" -1 and $+1$, respectively.

We denote the spinorial representation of the Lorentz group by:

$$\begin{pmatrix} \tilde{\xi}^{+} \\ \tilde{\xi}^{-} \end{pmatrix} = \begin{pmatrix} e^{\alpha/2} & 0 \\ 0 & e^{-\alpha/2} \end{pmatrix} \begin{pmatrix} \xi^{+} \\ \xi^{-} \end{pmatrix} \ , \tag{A.5}$$

such that the Dirac matrices, in the Majorana-Weyl representation, have the following form

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \ , \qquad \gamma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ . \tag{A.6}$$

To lower and rise spinor indices, we employ the following convention:

$$\xi^{\alpha} = \epsilon^{\alpha\beta}\xi_{\beta} \ , \qquad \xi_{\alpha} = \epsilon_{\alpha\beta}\xi^{\beta} \ , \tag{A.7}$$

where

$$\epsilon^{\alpha\beta} = \epsilon_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \ . \tag{A.8}$$

Chiral spinors are always defined on spaces of even dimensions through a matrix, γ_{d+1}, such that

$$(\gamma_{d+1})^2 = 1 \ , \qquad \{\gamma_{d+1}, \gamma_\mu\} = 0 \ . \tag{A.9}$$

We define projection operators by:

$$P_\pm = \frac{1 \pm \gamma_{d+1}}{2} \ , \tag{A.10}$$

such that chiral spinors are obtained by

$$\psi_\pm = \frac{1 \pm \gamma_{d+1}}{2} \psi \ , \tag{A.11}$$

where ψ_+ and ψ_- are independent spinors of Weyl. Majorana-Weyl spinors can only be defined in dimensions $d = 2 + 8n$, if there is a time-like dimension and $(D-1)$ space-like coordinates. They are chirals and satisfy the Majorana condition

$$\psi^* = \psi \ . \tag{A.12}$$

References

[1] L. Hörmander, "Fourier Integral Operators I," *Acta Math.* 127 (1971) 79;

[2] J.J. Duistermaat and L. Hörmander, "Fourier Integral Operators II," *Acta Math.* 128 (1972) 183;

[3] M.J. Radzikowski, "The Hadamard Condition and Kay's Conjecture in (Axiomatic) Quantum Field Theory on Curved Spacetime," Ph.D. dissertation, Princeton University, 1992;

[4] B.S. Kay, "Quantum Field Theory on Curved Space-Time," in Differential Geometrical Methods in Theoretical Physics, Dordrecht: Kluwer Academic Publishers, eds. K. Bleuler and M. Werner, 1988, pp. 373–393;

[5] B.S. DeWitt and R.W. Brehme, "Radiation Damping in a Gravitational Field," *Ann. Phys.* 9 (1960) 220;

[6] S.A. Fulling, "Aspects of Quantum Field Theory in Curved Space-Time," Cambridge University Press, 1989;

[7] B.S. Kay and R.M. Wald, "Theorems on the Uniqueness and Thermal Properties of Stationary, Nonsingular, Quasifree States on Spacetimes with Bifurcate Killing Horizon," *Phys. Rep.* 207 (1991) 49;

[8] R.M. Wald, "Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics," The University of Chicago Press, 1994;

[9] M. Köhler, "The Stress Energy Tensor of a Locally Supersymmetric Quantum Field on a Curved Spacetime," Doctoral dissertation, University of Hamburg, 1995, gr-qc/9505014; "New Examples for Wightman Fields on a Manifold," *Class. Quantum Grav.* 12 (1995) 1413;

[10] W. Junker, "Adiabatic Vacua and Hadamard States for Scalar Quantum Fields on Curved Spacetime," Doctoral dissertation, University of Hamburg, 1995, hep-th/9507097;

[11] B.S. Kay, M.J. Radzikowski and R.M. Wald, "Quantum Field Theory on Spacetimes with a Compactly Generated Cauchy-Horizon," *Commun. Math. Phys.* 183 (1997) 533;

[12] R. Brunetti, K. Fredenhagen and M. Köhler, "The Microlocal Spectrum Condition and Wick Polynomials of Free Fields in Curved Spacetimes," *Commun. Math. Phys.* 180 (1996) 663;

[13] R. Brunetti and K. Fredenhagen, "Interacting Quantum Fields in Curved Space: Renormalizability ofφ^4," gr-qc/9701048; "Interacting Quantum Fields on a Curved Background," hep-th/9709011; "Microlocal Analysis and Interacting Quantum Field Theories: Renormalization on Physical Backgrounds," *Commun. Math. Phys.* 208 (2000) 623;

[14] R. Verch, "Wavefront Sets in Algebraic Quantum Field Theory," *Commun. Math. Phys.* 205 (1999) 337; "On Generalizations of the Spectrum Condition," math-ph/0011026;

[15] E. Witten, "Supersymmetry and Morse Theory," *J. Diff. Geom.* 17 (1982) 661;

[16] A. Jaffe, A. Lesniewski and K. Osterwalder, "Quantum K-Theory I: The Chern Character," *Commun. Math. Phys.* 178 (1987) 313;

[17] A. Salam and J. Strathdee, "Supergauge Transformations," *Nucl. Phys. B* 76 (1974) 477;

[18] K. Osterwalder, "Supersymmetry Quantum Field Theory," Workshop on Constructive Results in Field Theory and Statistical Mechanics and Condensed Matter Physics, Palaiseau, France, July 1994, Springer LNP 446;

[19] M. Reed and B. Simon, "Fourier Analysis, Self-Adjointness," Academic Press, 1975;

[20] F. Treves, "Introduction to Pseudodifferential and Fourier Integral Operators," vols. 1,2, Plenum Press, New York, 1980;

[21] M.E. Taylor, "Pseudodifferential Operators," Princeton University Press, 1981;

[22] L. Hörmander, "The Analysis of Linear Partial Differential Operators I," Springer Verlag, Second Edition, 1990;

[23] A. Grigis and J. Sjöstrand, "Microlocal Analysis for Differential Operators," London Mathematical Society, Lecture Note Series 196, Cambridge University Press, 1994;

[24] M. Sato, "Hyperfunctions and Partial Differential Equations," Conf. on Funct. Anal. and Related Topics (1969) 31;

[25] D. Iagolnitzer, "Microlocal Essential Support of a Distribution and Decomposition Theorems – An Introduction," in Hyperfunctions and Theoretical Physics, Springer LNM 449 (1975) 121;

[26] J. Sjöstrand, "Singularités Analytiques Microlocales," Astérisque, 95 (1982);

[27] J.J. Kohn and L. Nirenberg, "On the Algebra of Pseudo-Differential Operators," *Comm. Pure Appl. Math* 18 (1965) 269;

[28] S.J. Gates, Jr., "Ectoplasm has no Topology," *Nucl. Phys. B* 541 (1999) 615;

[29] N. Dencker, "On the Propagation of Polarization Sets for Systems of Real Principal Type," *J. Funct. Anal.* 104 (1982) 414;

[30] S. Hollands, "The Hadamard Condition for Dirac Fields and Adiabatic States on Robertson-Walker Spacetimes," gr-qc/9906076;

[31] K. Kratzert, "Singularity structure of the Two Point Function of the Free Dirac Field on a Globally Hyperbolic Spacetime," *Annalen Phys.* 9 (2000) 475;

[32] H. Sahlmann and R. Verch, "Microlocal Spectrum Condition and Hadamard Form for Vector-Valued Quantum Fields in Curved Spacetime,"math-ph/0008029;

[33] O. Piguet and K. Sibold, "Renormalized Supersymmetry: The Perturbative Theory of $N = 1$ Supersymmetry Theories in Flat Space-Time," Progress in Physics, vol 12, Birkhäuser, 1986;

[34] S.J. Gates, Jr, M.T. Grisaru, M. Roček and W. Siegel, "Supersapce," Addison-Wesley, 1983;

[35] M.J. Duff, "Kaluza-Klein Theory in Perspective," hep-th/9410046;
J.M. Overduin and P.S. Wesson, "Kaluza-Klein Gravity," gr-qc/9805018;

[36] M.Green, J.H. Schwarz and E. Witten, "Superstring Theory," Cambridge University Press, 1987;
J. Polchinski, "String Theory," Cambridge University Press, 1998;

[37] J. Polchinski, "Tasi Lectures on D-Branes," hep-th/9611050; "Notes on D-Branes," hep-th/9602052;

[38] J.C. Baez, "An Introduction to Spin Foam Models of Quantum Gravity and BF Theory," gr-qc/9905087; "Spin Foam Perturbation Theory," gr-qc/9910050;

[39] C. Rovelli, "Loop Quantum Gravity," gr-qc/9710008;
A. Perez and C. Rovelli, "Spin Foam Model without Bubble Divergences," gr-qc/0006107; "Spin Foam Model for Lorentzian General Relativity," gr-qc/0009021;

[40] S.W. Hawking, "Particle Creation by Black Holes," *Commun. Math. Phys.* 43 (1975) 199;

[41] R.M. Wald, "The Back Reaction Effect in Particle Creation in Curved Spacetime," *Commun. Math. Phys.* 54 (1977) 1;

[42] D.H.T. Franco, J.A. Helayël-Neto and C.M.M. Polito, "The Hadamard Condition to the Wess-Zumino Model in Superspace," work in progress;

[43] D.H.T. Franco, J.A. Helayël-Neto and C.M.M. Polito, "Interacting Supersymmetric Quantum Fields in Curved Superspace: Renormalizability of the Wess-Zumino Model," work in progress.